DEPARTMENTS.

NOTE. All solutions of problems, problems for solution, and other department contributions should be sent direct to THE AMERICAN MATHEMATICAL MONTHLY, 1227 Clay Street, Springfield, Mo.

SOLUTIONS OF PROBLEMS.

NOTE. The following problems were received too late for publication: Calculus No. 200, and Geometry No. 263, solved by G. W. Greenwood. Credit is also given to J. Scheffer for solutions of Calculus No. 201, Diophantine Analysis No. 127, and Geometry No. 262.

ALGEBRA.

237. Proposed by F. P. MATZ, Sc. D., Ph. D., Reading, Pa.

Solve $x^2+y+z=12$.....(1) ; $x+y^2+z=8$.....(2) ; $x+y+z^2=6$.....(3).

Solution by L. E. NEWCOMB, Los Gatos, Cal.

Since (3) from (1) gives $x^2-x=6+z^2-z$, $\therefore\ x=\frac{1}{2}\pm\sqrt{(6\frac{1}{4}+z^2-z)}$.....(4); (3) from (2) gives $y=\frac{1}{2}\pm\sqrt{(2\frac{1}{4}+z^2-z)}$..........(5).

Substitute these values for x and y in (3); then $\frac{1}{2}+\sqrt{(6\frac{1}{4}+z^2-z)}+\frac{1}{2}+\sqrt{(2\frac{1}{4}+z^2-z)}+z^2=6$.

Whence $[\sqrt{(6\frac{1}{4}+z^2-z)}+\sqrt{(2\frac{1}{4}+z^2-z)}]^2=(5-z^2)^2$......... (6).

After expansion and transposition, (6) becomes $2\sqrt{[(6\frac{1}{4}+z^2-z)(2\frac{1}{4}+z^2-z)]}=z^4-12z^2+2z+16\frac{1}{2}$. Square both numbers; then $z^8-24z^6+4z^5+173z^4-40z^3-430z^2+100z+216=0$. The roots are $z=+1$, $+.204923$, $+2.39427$, $+3.48865$, -3.806118, -2.36109, -2.09446, $-.67044$.

Similarly two equations, one involving x, the other, y, are derived; or the values of x, y may be found from (4), (5), respectively. These values are:

$x=3$, $+3.3983$, -2.59649, -3.3642, -4.45405, -3.2664, $+4.06808$, $+3.21476$;

$y=2$, -1.5976, $+2.86394$, -2.80636, -4.0324, $+3.69155$, -2.4548, $+2.33574$.

Also solved by A. H. Holmes, J. Scheffer, G. B. M. Zerr, and the Proposer.

238. Proposed by S. A. COREY, Hiteman, Iowa.

Prove that $\dfrac{1}{n+1}+\dfrac{1}{3(n+3)}+\dfrac{1}{5(n+5)}+..........=\frac{1}{2}\left[\dfrac{1}{(n-1)}+\dfrac{1}{3(n-3)}+\dfrac{1}{5(n-5)}+.....+\dfrac{1}{l(n-l)}\right]$, n being an even positive integer and $l=n-1$.

Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Putting successively $m=2$, 4, 6, 8, ...$-m$, we get

$$\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+\text{.........ad inf.}=\tfrac{1}{2}[(\tfrac{1}{1}-\tfrac{1}{3})+(\tfrac{1}{3}-\tfrac{1}{5})+(\tfrac{1}{5}-\tfrac{1}{7})+.........]=\tfrac{1}{2}(1)=\tfrac{1}{2}.$$